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SEGMENTED SENSORS AND ACTUATORS FOR THICK PLATES AND SHELLS PART I: ANALYSIS USING FSDT

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This paper presents modal sensitivity factors, actuation factors and controlled damping ratio for segmented distributed piezoelectric sensor and actuator layers laminated on simply supported thick rectangular plates and circular cylindrical shells made of cross-ply composite laminate. A Flugge-type first order shear deformation theory (FSDT) has been developed including rotary inertia of the elastic plies and inertia, stiffness and piezoelectric effects (direct and converse) of the piezoelectric layers. Plates and shells with any cross-ply lamination scheme (symmetric/unsymmetric) are considered with the piezoelectric sensor and actuator layers rectangularly segmented. The Navier-type solution is obtained for the case of velocity feedback using modal filtering. Analytical expressions for modal membrane and bending sensitivity factors, membrane and bending actuation factors and controlled damping ratio are developed for cylindrical shells using FSDT and classical lamination theory. The contribution of the in-plane displacement components is included. The results for plates are obtained as a particular case of the results for circular cylindrical shells.

1. INTRODUCTION

Composite plates and shells with piezoelectric layers for sensing and actuation are being used in smart structures where shape control is important [1–3]. By proper placement, distributed sensors can be used to sense membrane response or bending response or both. The sensing and actuation effects of distributed piezoelectric sensors and actuators depend on their shape, thickness, material properties, placement, spatial shaping and spatial distribution. There are observability and controllability deficiencies in monitoring and control of plates and shells when a single piece fully distributed piezoelectric sensor/actuator is used [3]. Spatial shaping of distributed sensors and actuators, for example by segmenting them into a number of smaller pieces, can improve the controllability and observability. The effectiveness of piezoelectric distributed sensors and actuators, segmented into rectangular patches, laminated on composite simply supported rectangular thin plates and circular cylindrical thin shell panels has been investigated by Tzou and Fu [4,5] and Tzou *et al.* [6,7]. Classical lamination theory (CLT) has been applied neglecting shear and rotary inertia.

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The effect of transverse shear is significant for composite thin plates and shells with fibre-reinforced composite elastic substrate, even for lower modes due to the small ratio of the transverse shear modulus to the longitudinal Young's modulus. Even for the case of isotropic elastic core, the shear deformation and rotary inertia effects are significant for higher modes. This work presents modal sensitivity and actuation factors and controlled modal damping ratio for rectangularly segmented piezoelectric sensor and actuator layers laminated on simply supported rectangular plates and circular cylindrical shells made of cross-ply composite laminate using a first order shear deformation theory (FSDT). The FSDT has been developed for cylindrical shells based on Flugge's approximations. The rotary inertia of the elastic plies and inertia, stiffness and piezoelectric effects (direct and converse) of the piezoelectric layers are included. Analytical expressions for modal membrane and bending sensitivity factors, membrane and bending actuation factors and controlled damping ratio are developed for the case of velocity feedback using Navier-type solution including the contribution of the in-plane displacement components. The results for plates are obtained as a particular case of the results for circular cylindrical shells. It has been observed from the numerical results that the modal sensitivity and actuation factors are overpredicted by CLT in comparision to FSDT. Hence the amplification factor required for feedback control based on CLT needs to be suitably modified.

2. GOVERNING EQUATIONS FOR FSDT

Consider a finite, simply supported, composite circular cylindrical shell panel of span angle ψ (Figure 1) made up of a cross-ply laminated composite elastic substrate with surface bonded piezoelectric layers of orthorhombic crystal class mm2 with poling in the z direction. FSDT is developed based on Flugge's shell theory approximations. Let u^0, v^0, w^0 be the displacements of the mid-surface in the cylindrical co-ordinates x, θ, z, and ψ_1, ψ_2 be the rotations of its normal. The displacements u, v, w, in FSDT are approximated as

$$u(x,\theta,z,t)=u^0(x,\theta,t)+z\psi_1(x,\theta,t),\quad v(x,\theta,z,t)=v^0(x,\theta,t)+z\psi_2(x,\theta,t),$$

$$w(x,\theta,z,t)=w^0(x,\theta,t). \tag{1}$$

The strain–displacement relations yield the strains as

$$\varepsilon_x=u^0_{,x}+z\psi_{1,x},\quad \varepsilon_\theta=(v^0_{,\theta}+z\psi_{2,\theta}+w^0)/(R+z),\quad \varepsilon_z=0,\quad \gamma_{zx}=\psi_1+w^0_{,x},$$

$$\gamma_{\theta z}=\psi_2+(w^0_{,\theta}-v^0-z\psi_2)/(R+z),\quad \gamma_{x\theta}=v^0_{,x}+(u^0_{,\theta}+z\psi_{1,\theta})/(R+z)+z\psi_{2,x}. \tag{2}$$

For piezoelectric layers, stress σ, strain ε, electric field E and electric displacement D are related by

$$\varepsilon=S\sigma+d^{\mathrm{T}}E,\quad D=e\varepsilon+\eta E, \tag{3}$$

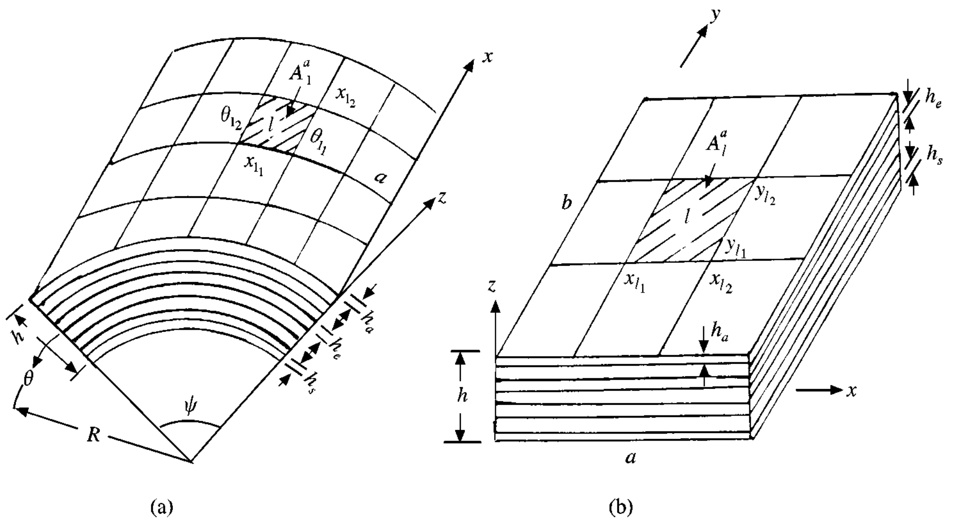

Figure 1. Geometry of composite shell and plate.

where the superscript T denotes matrix transpose and S, d, e, η are the matrices of elastic compliance, piezoelectric strain constants, piezoelectric stress constants and permittivities with

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix}, \quad e = dS^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix},$$

$$\eta = \begin{bmatrix} \eta_{11} & 0 & 0 \\ 0 & \eta_{22} & 0 \\ 0 & 0 & \eta_{33} \end{bmatrix}, \tag{4}$$

Let $E_z = E_z^d + E_z^a$, where E_z^d, and E_z^a are the electric fields due to the direct piezoelectric effect and due to applied actuation potentials. In the constitutive equation for D_z, obtained from equation (3),

$$D_z = [e_{31}\varepsilon_x + e_{32}\varepsilon_\theta + \eta_{33}E_z^d] + \eta_{33}E_z^a, \tag{5}$$

the bracketted expression is equated to zero as its value is very small compared to the last term [6]. Hence

$$E_z^d = -[e_{31}\varepsilon_z + e_{32}\varepsilon_\theta]/\eta_{33}, \quad E_z = E_z^a - [e_{31}\varepsilon_x + e_{32}\varepsilon_\theta]/\eta_{33}. \tag{6}$$

Assuming $\sigma_z \simeq 0$, neglecting E_x^d, E_θ^d compared to E_x^a, E_θ^a [6] and using equation (6) in equation (3) yield

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{bmatrix} - \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix} E_z^a, \quad \begin{bmatrix} \tau_{\theta z} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} Q_{44}\gamma_{\theta z} - e_{24}E_\theta^a \\ Q_{55}\gamma_{zx} - e_{15}E_x^a \end{bmatrix} \tag{7}$$

with

$$\bar{Q}_{\alpha\beta} = Q_{\alpha\beta} + e_{3\alpha}e_{3\beta}/\eta_{33}, \quad Q_{44} = g_{23}, \quad Q_{55} = g_{31}, \quad Q_{66} = g_{12},$$

$$Q_{11} = Y_1/(1 - v_{12}v_{21}), \quad Q_{12} = v_{12}Y_2/(1 - v_{12}v_{21}), \quad Q_{22} = Y_2/(1 - v_{12}v_{21}),$$

$$e_{31} = Q_{11}d_{31} + Q_{12}d_{32}, \quad e_{32} = Q_{12}d_{31} + Q_{22}d_{32}, \quad e_{24} = Q_{44}d_{24}, \quad e_{15} = Q_{55}d_{15},$$

where Y_i are Young's moduli, v_{ij} the Poisson ratios and g_{ij} the shear moduli.

For the mid-surface, the six force (N_x, N_θ, $N_{x\theta}$, $N_{\theta x}$, Q_x, Q_θ) and four moment (M_x, M_θ, $M_{x\theta}$, $M_{\theta x}$) resultants per unit length, and the distributed load consisting of three forces (p_x, p_θ, p_z) and two moments (m_x, m_θ) per unit area, are defined in terms of stresses by

$$\begin{bmatrix} N_x, N_\theta, N_{x\theta}, N_{\theta x} \\ M_x, M_\theta, M_{x\theta}, M_{\theta x} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} 1 \\ z \end{bmatrix} [\sigma_x(1 + z/R),\ \sigma_\theta,\ \tau_{x\theta}(1 + z/R),\ \tau_{\theta x}]\, \mathrm{d}z,$$

$$\begin{bmatrix} Q_x \\ Q_\theta \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \tau_{xz}(1 + z/R) \\ \tau_{\theta z} \end{bmatrix} \mathrm{d}z, \tag{8}$$

$$(p_x, p_\theta, p_z) = [(1 + z/R)(\tau_{xz}, \tau_{z\theta}, \sigma_z)]|_{-h/2}^{h/2}, \quad (m_x, m_\theta) = [(1 + z/R)z(\tau_{zx}, \tau_{z\theta})]|_{-h/2}^{h/2}. \tag{9}$$

Using expressions (2) and (7) in equation (8), approximating $(1 + z/R)^{-1} \simeq 1 - z/R + z^2/R^2$, and retaining terms upto order z^2/R^2 in the integrands yield the following relations between the resultants and displacements:

$$N_x = (\bar{A}_{11} + \bar{B}_{11}/R)u^0_{,x} + \bar{A}_{12}(v^0_{,\theta} + w^0)/R + (\bar{B}_{11} + \bar{D}_{11}/R)\psi_{1,x}$$
$$+ \bar{B}_{12}\psi_{2,\theta}/R + N^a_x,$$

$$N_\theta = \bar{A}_{12}u^0_{,x} + (\bar{A}_{22} - \bar{B}_{22}/R + \bar{D}_{22}/R^2)(v^0_{,\theta} + w^0)/R + \bar{B}_{12}\psi_{1,x}$$
$$+ (\bar{B}_{22} - \bar{D}_{22}/R)\psi_{2,\theta}/R + N^a_\theta.$$

$$N_{x\theta} = A_{66}u^0_{,\theta}/R + (A_{66} + B_{66}/R)v^0_{,x} + B_{66}\psi_{1,\theta}/R + (B_{66} + D_{66}/R)\psi_{2,x},$$

$$N_{\theta x} = (A_{66} - B_{66}/R + D_{66}/R^2)u^0_{,\theta}/R + A_{66}v^0_{,x} + (B_{66} - D_{66}/R)\psi_{1,\theta}/R$$
$$+ B_{66}\psi_{2,x}.$$

$$M_x = (\bar{B}_{11} + \bar{D}_{11}/R)u^0_{,x} + \bar{B}_{12}(v^0_{,\theta} + w^0)/R + \bar{D}_{11}\psi_{1,x} + \bar{D}_{12}\psi_{2,\theta}/R + M^a_x,$$

$$M_\theta = \bar{B}_{12}u^0_{,x} + (\bar{B}_{22} - \bar{D}_{22}/R)(v^0_{,\theta} + w^0)/R + \bar{D}_{12}\psi_{1,x} + \bar{D}_{22}\psi_{2,\theta}/R + M^a_\theta.$$

$$M_{x\theta} = B_{66}u^0_{,\theta}/R + (B_{66} + D_{66}/R)v^0_{,x} + D_{66}\psi_{1,\theta}/R + D_{66}\psi_{2,x},$$

$$M_{\theta x} = (B_{66} - D_{66}/R)u^0_{,\theta}/R + B_{66}v^0_{,x} + D_{66}\psi_{1,\theta}/R + D_{66}\psi_{2,x}.$$

$$Q_x = (A_{55} + B_{55}/R)(\psi_1 + w^0_{,x}) + Q^a_x,$$

$$Q_\theta = (A_{44} - B_{44}/R + D_{44}/R^2)[\psi_2 + (w^0_{,\theta} - v^0)/R] + Q^a_\theta \tag{10}$$

with

$$\begin{bmatrix} N^a_x \\ M^a_x \end{bmatrix} = -\int_{-h/2}^{h/2} (1 + z/R) \begin{bmatrix} e_{31}E^a_z \\ ze_{31}E^a_z \end{bmatrix} \mathrm{d}z, \quad \begin{bmatrix} N^a_\theta \\ M^a_\theta \end{bmatrix} = -\int_{-h/2}^{h/2} \begin{bmatrix} e_{32}E^a_z \\ ze_{32}E^a_z \end{bmatrix} \mathrm{d}z, \tag{11}$$

$$\begin{bmatrix} Q^a_x \\ Q^a_\theta \end{bmatrix} = -\int_{-h/2}^{h/2} \begin{bmatrix} k_{55}(1 + z/R)e_{15}E^a_x \\ k_{44}e_{24}E^a_\theta \end{bmatrix} \mathrm{d}z,$$

$$\begin{bmatrix} A_{ij} & B_{ij} & D_{ij} \\ \bar{A}_{ij} & \bar{B}_{ij} & \bar{D}_{ij} \end{bmatrix} = \int_{-h/2}^{h/2} k^2_{ij} \begin{bmatrix} Q_{ij} \\ \bar{Q}_{ij} \end{bmatrix} [1\ z\ z^2]\, \mathrm{d}z, \tag{12}$$

and $k_{ij} = 1$ except for k_{44}, k_{55}. As in reference [8], the shear correction factors are taken as $k_{44} = k_{55} = (5/6)^{1/2} = 0{\cdot}91287$.

Defining $[\rho_0 h\ \rho_1 h^2\ \rho_2 h^3] = \int [1\ z\ z^2] \rho(1 + z/R)\, \mathrm{d}z$, the equations of motion of the panel:

$$N_{x,x} + N_{\theta x,\theta}/R + p_x = \rho_0 h\ddot{u}^0 + \rho_1 h^2 \ddot{\psi}_1,$$

$$(Q_\theta + N_{\theta,\theta})/R + N_{x\theta,x} + p_\theta = \rho_0 h\ddot{v}^0 + \rho_1 h^2 \ddot{\psi}_2,$$

$$Q_{x,x} + (Q_{\theta,\theta} - N_\theta)/R + p_z = \rho_0 h\ddot{w}^0,$$

$$M_{x,x} + M_{\theta x,\theta}/R - Q_x + m_x = \rho_1 h^2 \ddot{u}^0 + \rho_2 h^3 \ddot{\psi}_1,$$

$$M_{\theta,\theta}/R + M_{x\theta,x} - Q_\theta + m_\theta = \rho_1 h^2 \ddot{v}^0 + \rho_2 h^3 \ddot{\psi}_2, \tag{13}$$

yield the following equations of motion in terms of displacements, using relations (10):

$$(\bar{A}_{11} + \bar{B}^*_{11})u^0_{,xx} + (A_{66} - B^*_{66} + D^*_{66})u^0_{,\theta\theta}/R^2 + (\bar{A}_{12} + A_{66})v^0_{,x\theta}/R + \bar{A}_{12}w^0_{,x}/R$$

$$+ (\bar{B}_{11} + \bar{D}_{11}/R)\psi_{1,xx} + (B^*_{66} - D^*_{66})\psi_{1,\theta\theta}/R + (\bar{B}^*_{12} + B^*_{66})\psi_{2,x\theta} + P_1$$

$$= \rho_0 h\ddot{u}^0 + \rho_1 h^2 \ddot{\psi}_1.$$

$$(\bar{A}_{12}+A_{66})u^0_{,x\theta}/R+(A_{66}+B^*_{66})v^0_{,xx}+(\bar{A}_{22}-\bar{B}^*_{22}+\bar{D}^*_{22})v^0_{,\theta\theta}/R^2$$

$$+(B^*_{44}-D^*_{44}-A_{44})v^0/R^2+(A_{44}-B^*_{44}+D^*_{44}+\bar{A}_{22}-\bar{B}^*_{22}+\bar{D}^*_{22})w^0_{,\theta}/R^2$$

$$+(\bar{B}^*_{12}+B^*_{66})\psi_{1,x\theta}+(B_{66}+D_{66}/R)\psi_{2,xx}+(\bar{B}^*_{22}-\bar{D}^*_{22})\psi_{2,\theta\theta}/R$$

$$-(B^*_{44}-D^*_{44}-A_{44})\psi_2/R+P_2=\rho_0 h\ddot{v}^0+\rho_1 h^2\ddot{\psi}_2.$$

$$\bar{A}_{12}u^0_{,x}/R+(A_{44}-B^*_{44}+D^*_{44}+\bar{A}_{22}-\bar{B}^*_{22}+\bar{D}^*_{22})v^0_{,\theta}/R^2-(A_{55}+B^*_{55})w^0_{,xx}$$

$$+(B^*_{44}-D^*_{44}-A_{44})w^0_{,\theta\theta}/R^2+(\bar{A}_{22}-\bar{B}^*_{22}+\bar{D}^*_{22})w^0/R^2$$

$$+(\bar{B}^*_{12}-B^*_{55}-A_{55})\psi_{1,x}+(\bar{B}^*_{22}+B^*_{44}-A_{44}-\bar{D}^*_{22}-D^*_{44})\psi_{2,\theta}/R+P_3$$

$$=\rho_0 h\ddot{w}^0$$

$$(\bar{B}_{11}+\bar{D}_{11}/R)u^0_{,xx}+(B^*_{66}-D^*_{66})u^0_{,\theta\theta}/R+(\bar{B}^*_{12}+B^*_{66})v^0_{,x\theta}+(\bar{B}^*_{12}-B^*_{55}-A_{55})w^0_{,x}$$

$$+\bar{D}_{11}\psi_{1,xx}+D^*_{66}\psi_{1,\theta\theta}-(A_{55}+B^*_{55})\psi_1+(\bar{D}_{12}+D_{66})\psi_{2,x\theta}/R+P_4$$

$$=\rho_1 h^2\ddot{u}^0+\rho_2 h^3\ddot{\psi}_1,$$

$$(B^*_{12}+B^*_{66})u^0_{,x\theta}+(B_{66}+D_{66}/R)v^0_{,xx}+(\bar{B}^*_{22}-\bar{D}^*_{22})v^0_{,\theta\theta}/R$$

$$-(B^*_{44}-D^*_{44}-A_{44})v^0/R+(\bar{B}^*_{22}+B^*_{44}-A_{44}-\bar{D}^*_{22}-D^*_{44})w^0_{,\theta}/R$$

$$+(\bar{D}_{12}+D_{66})\psi_{1,x\theta}/R+(B^*_{44}-D^*_{44}-A_{44})\psi_2+P_5=\rho_1 h^2\ddot{v}^0+\rho_2 h^3\ddot{\psi}_2, \quad (14)$$

with

$$P_1=p_x+N^a_{x,x},\quad P_2=p_\theta+(Q^a_\theta+N^a_{\theta,\theta})/R,\quad P_3=p_z+Q^a_{x,x}+[Q^a_{\theta,\theta}-N^a_\theta]/R,$$

$$P_4=m_x-Q^a_x+M^a_{x,x},\quad P_5=m_\theta-Q^a_\theta+M^a_{\theta,\theta}/R, \quad (15)$$

$$\bar{B}^*_{ij}=\bar{B}_{ij}/R,\quad \bar{D}^*_{ij}=\bar{D}_{ij}/R^2\quad B^*_{ij}=B_{ij}/R,\quad D^*_{ij}=D_{ij}/R^2. \quad (16)$$

The boundary conditions at the simply supported ends are taken as

$$\text{at } x=0, a:\quad N_x=0,\quad v^0=w^0=0,\quad M_x=0,\quad \psi_2=0.$$

$$\text{at } \theta=0, \psi:\quad u^0=w^0=0,\quad N_\theta=0,\quad \psi_1=0,\quad M_\theta=0. \quad (17)$$

The Navier-type solutions of equations (14) satisfying equations (17) are expanded as

$$(u^0, \psi_1, Q_x, p_x, m_x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (u^0, \psi_1, Q_x, p_x, m_x)_{mn}(t) \cos \bar{m}x \sin \bar{n}\theta,$$

$$(v^0, \psi_2, Q_\theta, p_\theta, m_\theta) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (v^0, \psi_2, Q_\theta, p_\theta, m_\theta)_{mn}(t) \sin \bar{m}x \cos \bar{n}\theta,$$

$$(w^0, N_x, N_\theta, M_x, M_\theta, p_z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (w^0, N_x, N_\theta, M_x, M_\theta, p_z)_{mn}(t) \sin \bar{m}x \sin \bar{n}\theta,$$

$$(N_{x\theta}, N_{\theta x}, M_{x\theta}, M_{\theta x}) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (N_{x\theta}, N_{\theta x}, M_{x\theta}, M_{\theta x})_{mn}(t) \cos \bar{m}x \cos \bar{n}\theta, \quad (18)$$

with $\bar{m} = m\pi/a$ and $\bar{n} = n\pi/\psi$. Similar expansions are used for Q_x^a, Q_θ^a, N_x^a, N_θ^a, M_x^a, M_θ^a. Substitution of expansions (18) into equation (14) yields discretized equations:

$$M\ddot{U} + KU = H, \quad (19)$$

where M, K are the inertia and stiffness matrices and U, H are the displacement and load vectors for the (m, n)th Fourier component. The non-zero elements of the 5×5 symmetric matrices M, K and vectors U, H are given by

$$M_{11} = M_{22} = M_{33} = \rho_0 h, \quad M_{44} = M_{55} = \rho_2 h^3, \quad M_{14} = M_{25} = \rho_1 h^2,$$

$$K_{11} = (\bar{A}_{11} + \bar{B}^*_{11})\bar{m}^2 + (A_{66} - B^*_{66} + D^*_{66})\bar{n}^2/R^2, \quad K_{12} = (\bar{A}_{12} + A_{66})\bar{m}\bar{n}/R,$$

$$K_{13} = -\bar{A}_{12}\bar{m}/R,$$

$$K_{14} = (\bar{B}_{11} + \bar{D}_{11}/R)\bar{m}^2 + (B^*_{66} - D^*_{66})\bar{n}^2/R, \quad K_{23} = -(A_{44} - B^*_{44} + D^*_{44} + \bar{A}_{22} - \bar{B}^*_{22} + \bar{D}^*_{22})\bar{n}/R^2,$$

$$K_{22} = (A_{66} + B^*_{66})\bar{m}^2 + (\bar{A}_{22} - \bar{B}^*_{22} + \bar{D}^*_{22})\bar{n}^2/R^2 - (B^*_{44} - D^*_{44} - A_{44})/R^2,$$

$$K_{15} = (\bar{B}^*_{12} + B^*_{66})\bar{m}\bar{n},$$

$$K_{24} = (\bar{B}^*_{12} + B^*_{66})\bar{m}\bar{n}, \quad K_{25} = (B_{66} + D_{66}/R)\bar{m}^2 + (\bar{B}^*_{22} - \bar{D}^*_{22})\bar{n}^2/R + (B^*_{44} - D^*_{44} - A_{44})/R,$$

$$K_{33} = (A_{55} + B^*_{55})\bar{m}^2 - (B^*_{44} - D^*_{44} - A_{44})\bar{n}^2/R^2 + (\bar{A}_{22} - \bar{B}^*_{22} + \bar{D}^*_{22})/R^2,$$

$$K_{45} = (\bar{D}_{12} + D_{66})\bar{m}\bar{n}/R,$$

$$K_{35} = -(\bar{B}^*_{22} + B^*_{44} - A_{44} - \bar{D}^*_{22} - D^*_{44})\bar{n}/R, \quad K_{44} = \bar{D}_{11}\bar{m}^2 + D^*_{66}\bar{n}^2 + A_{55} + B^*_{55},$$

$$K_{34} = -(B^*_{12} - B^*_{55} - A_{55})\bar{m}, \quad K_{55} = D_{66}\bar{m}^2 + \bar{D}^*_{22}\bar{n}^2 - B^*_{44} + D^*_{44} + A_{44}.$$

$$U = [u^0\ v^0\ w^0\ \psi_1\ \psi_2]^{\mathrm{T}}_{mn}, \quad H_1 = \bar{m}(N^a_x)_{mn} + p_{x_{mn}},$$

$$H_2 = (Q^a_\theta + \bar{n}N^a_\theta)_{mn}/R + p_{\theta_{mn}}, \quad H_3 = p_{z_{mn}} - \bar{m}Q^a_{x_{mn}} - (\bar{n}Q^a_\theta + N^a_\theta)_{mn}/R,$$

$$H_4 = m_{x_{mn}} + \bar{m}(M^a_x)_{mn} - Q^a_{x_{mn}}, \quad H_5 = m_{\theta_{mn}} - Q^a_{\theta_{mn}} + \bar{n}(M^a_\theta)_{mn}/R. \tag{20}$$

For undamped, free, synchronous vibrations at natural frequency ω, with $U = X\cos\omega t$, equation (19) yields $KX = \omega^2 MX$. The five natural frequencies and mode shapes for FSDT correspond to two in-plane, two thickness-shear and one bending mode of vibration. Let X^b be the bending mode, with natural frequency ω, normalized with respect to the inertia matrix M: $X^{b\mathrm{T}}MX^b = 1$. For this (m, n)th bending mode of vibration, let $U(t) = \zeta(t)X^b$. The governing equation of motion for this mode can be derived from equation (19) as

$$\ddot{\zeta} + 2\xi\omega\dot{\zeta} + \omega^2\zeta = F, \tag{21}$$

where $F = X^{b^{\mathrm{T}}}H$, and passive damping has been included in terms of the modal damping ratio ξ.

3. MODAL SENSITIVITY AND ACTUATION FACTORS FOR SHELLS USING FSDT

Consider that the piezoelectric sensor layer on the inside and the piezoelectric actuator layer on the outside are divided into equal patches with p segments in the x-direction and q segments in the θ direction (Figure 1). Let their thicknesses be h_s, h_a and the z-co-ordinate of their mid-surfaces be z_s, z_a respectively. The electrodes are set up over each patch separately. The interfaces of the piezoelectric sensor and actuator layers with the elastic substrate are earthed.

Using equation (6) with $E^a_z = 0$ and integrating $E_z = -\phi_{,z}$ across the sensor thickness, the potential ϕ at the inside of the sensor layer is obtained as

$$\phi = \int_{h_s} E_z\,\mathrm{d}z = -\int_{h_s} (e_{31}\varepsilon_x + e_{32}\varepsilon_\theta)\eta^{-1}_{33}\,\mathrm{d}z. \tag{22}$$

The contribution $\phi^{s(b)}_{mn_l}$ to the averaged sensed potential $\phi^{s(b)}_{mn}$ for the (m, n)th bending mode $U(t) = \zeta(t)X^b$, due to an electrode over a sensor patch l of area A^s_l extending from x_{l_1} to x_{l_2} and θ_{l_1} to θ_{l_2}, is obtained from equation (22) using expansions (18) and strain–displacement relations (2):

$$\phi^{s(b)}_{mn_l} = \frac{1}{A^s_l}\int_{A^s_l}\int_{h_s} [\bar{m}e_{31}(X^b_1 + zX^b_4) + e_{32}\{\bar{n}(X^b_2 + zX^b_5) - X^b_3\}/$$

$$\times (R + z)]\eta^{-1}_{33}\,\mathrm{d}z\,\zeta\sin\bar{m}x\sin\bar{n}\theta\,\mathrm{d}A$$

with $dA = dx\, R_1\, d\theta$, where R_1 is the radius of the sensing surface. The integral is evaluated exactly and the resulting logarithmic term $\ln[(R + z_s + h_s/2)/(R + z_s - h_s/2)]$ is expanded as $[h_s/(R + z_s)][1 + h_s^2/12(R + z_s)^2]$ to yield

$$\phi_{mn_l}^{s(b)} = \zeta(S_{mn}^m + S_{mn}^b)(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2})(\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2})aR_1\psi/A_l^s \tag{23}$$

with

$$S_{mn}^m = h_s[\bar{m}e_{31}X_1^b + e_{32}(\bar{n}X_2^b - X_3^b)\{1 + h_s^2/12(R + z_s)^2\}/(R + z_s)]/(\pi^2 mn\eta_{33}), \tag{24}$$

$$S_{mn}^b = h_s[\bar{m}z_s e_{31}X_4^b + \bar{n}e_{32}\{z_s(R + z_s) - Rh_s^2/12(R + z_s)^3\}X_5^b]/(\pi^2 mn\eta_{33}). \tag{25}$$

where S_{mn}^m, S_{mn}^b are the membrane and bending modal sensitivities for the (m, n)th bending mode. S_{mn}^m, S_{mn}^b are independent of time and location of the sensor patch. The total modal sensitivity S_{mn}^t due to the direct elastic-piezoelectric effect for this mode is given by $S_{mn}^t = S_{mn}^m + S_{mn}^b$. The term $\zeta(t)$ represents the time effect and $(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2})(\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2})aR_1\psi/A_l^s$ represents the spatial effect of the area and location of the electrode patch. The sensor signal depends on the modal sensitivity, modal participation factor and spatial distribution. Hence, the total modal signal for the (m, n)th bending mode is obtained as $\phi_{mn}^{s(b)} = \sum_{l=1}^{pq} \phi_{mn_l}^{s(b)}$.

Consider the case of damped free vibration, i.e., $p_x = p_y = p_z = 0$, $m_x = m_y = 0$. Let a constant actuation potential ϕ^l be applied to an electrode over the actuator patch l of area A_l^a extending from x_{l_1} to x_{l_2} θ_{l_1} to θ_{l_2}. The potential ϕ^a is approximated to vary linearly through the thickness of the actuator. Hence, for $z_a - h_a/2 \leqslant z \leqslant z_a + h_a/2$,

$$\phi^a = [(z - z_a + h_a/2)/h_a]\phi^l[u(x - x_{l_1}) - u(x - x_{l_2})][u(\theta - \theta_{l_1}) - u(\theta - \theta_{l_2})],$$

$$E_x^a = -[(z - z_a + h_a/2)/h_a]\phi^l[\delta(x - x_{l_1}) - \delta(x - x_{l_2})][u(\theta - \theta_{l_1}) - u(\theta - \theta_{l_2})],$$

$$E_\theta^a = -[(z - z_a + h_a/2)/h_a]\phi^l[u(x - x_{l_1}) - u(x - x_{l_2})] \times [\delta(\theta - \theta_{l_1}) - \delta(\theta - \theta_{l_2})]/(R + z),$$

$$E_z^a = \phi^l[u(x - x_{l_1}) - u(x - x_{l_2})][u(\theta - \theta_{l_1}) - u(\theta - \theta_{l_2})]/h_a. \tag{26}$$

where $u(x)$ is the unit step function and $\delta(x)$ is the Dirac-delta function. Using equations (26), equations (11), (12) and (18) yield the following Fourier components of the force resultants due to actuation

$$[N_x^a, N_\theta^a]_{mn} = \frac{4}{a\psi\bar{m}\bar{n}}[e_{31}(1 + z_a/R), e_{32}]\phi^l(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2})(\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2}),$$

$$[M_x^a, M_\theta^a]_{mn} = \frac{4}{a\psi\bar{m}\bar{n}}[e_{31}\{z_a(1 + z_a/R) + h_a^2/12R\}, z_a e_{32}]\phi^l(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2}) \times (\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2}),$$

$$Q^a_{x_{mn}} = -\frac{2}{a\psi\bar{n}} k_{55} h_a e_{15}[1 + h_a/6R + z_a/R]\phi^l(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2})$$

$$\times(\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2}),$$

$$Q^a_{\theta_{mn}} = -\frac{2}{a\psi\bar{m}} k_{44}\left[\frac{h_a}{R + z_a} - \frac{h_a^3}{6(R + z_a)^2} + \frac{h_a^3}{12(R + z_a)^3}\right] e_{24}\phi^l(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2})$$

$$\times(\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2}). \tag{27}$$

Using equation (20) for Q and equation (27), the contribution F_l of the actuator patch l to the modal load $F = X^{b\mathrm{T}}H = \sum_{l=1}^{pq} F_l$, for the (m, n)th bending mode, is given by

$$F_l = -A^t_{mn}\phi^l(\cos\bar{m}x_{l_1} - \cos\bar{m}x_{l_2})(\cos\bar{n}\theta_{l_1} - \cos\bar{n}\theta_{l_2}) \tag{28}$$

with

$$A^t_{mn} = A^m_{mn} + A^b_{mn},$$

$$A^m_{mn} = -\frac{4}{aR\psi}\left[Re_{31}(1 + z_a/R)X^b_1/\bar{n} + \left\{e_{32} - \frac{1}{2}k_{44}e_{24}\left\{\frac{h_a}{R + z_a} - \frac{h_a^2}{6(R + z_a)^2}\right.\right.\right.$$

$$\left.\left.\left. + \frac{h_a^3}{12(R + z_a)^3}\right\}X^b_2/\bar{m} - e_{32}X^b_3/\bar{m}\bar{n}\right],$$

$$A^b_{mn} = -\frac{4}{aR\psi}\left[\frac{1}{2}h_a\left\{k_{55}e_{15}R(1 + h_a/6R + z_a/R)\bar{m}/\bar{n} + \bar{n}k_{44}e_{24}\left\{\frac{1}{R + z_a} - \frac{h_a}{6(R + z_a)^2}\right.\right.\right.$$

$$\left.\left. + \frac{h_a^2}{12(R + z_a)^3}\right\}\Big/\bar{m}\right\}X^b_3 + \{e_{31}(z_a(1 + z_a/R) + h_a^2/12R)$$

$$+ \tfrac{1}{2}h_a k_{55}e_{15}(1 + h_a/6R + z_a/R)\}X^b_4 R/\bar{n}$$

$$\left. + \left\{z_a e_{32} + \frac{1}{2}h_a k_{44}e_{24}R\left\{\frac{1}{R + z_a} - \frac{h_a}{6(R + z_a)^2} + \frac{h_a^2}{12(R + z_a)^3}\right\}\right\}X^b_5/\bar{m}\right]. \tag{29}$$

A^t_{mn} is the total (m, n)th modal actuation factor which is independent of time and spatial distribution. It has been divided into the membrane actuation factor A^m_{mn} and the bending actuation factor A^b_{mn}, respectively, representing the membrane and bending control actions. The modal control force depends on the the spatial distribution, the modal actuation factor and the voltages applied to the actuator patches.

4. MODAL SENSITIVITY AND ACTUATION FACTORS FOR SHELLS USING CLT

In the classical lamination theory (CLT) of thin shells, we assume that the transverse shear strains $\gamma_{zx} = \gamma_{\theta z} = 0$, i.e., $\psi_1 = -w^0_{,x}$, $\psi_2 = (v^0 - w^0_{,\theta})R$ and terms involving $\rho_1 h$, $\rho_2 h^2$ are neglected. The equations of motion for the CLT are

$$N_{x,x} + N_{\theta x,\theta}/R + p_x = \rho_0 h\ddot{u}^0, \quad N_{\theta,\theta}/R + N_{x\theta,x} + M_{\theta,\theta}/R^2 + M_{x\theta,x}/R + m_\theta/R$$
$$+ p_\theta = \rho_0 h\ddot{v}^0, \quad M_{x,xx} + (M_{\theta x} + M_{x\theta})_{,x\theta}/R + M_{\theta,\theta\theta}/R^2 - N_\theta/R + m_{x,x}$$
$$+ m_{\theta,\theta}/R + p_z = \rho_0 h\ddot{w}^0. \tag{30}$$

Proceeding as in FSDT, these can be expressed in terms of displacements, u^0, v^0, w^0, as

$$(\bar{A}_{11} + \bar{B}^*_{11})u^0_{,xx} + (A_{66} - B^*_{66} + D^*_{66})u^0_{,\theta\theta}/R^2 + (\bar{A}_{12} + A_{66} + \bar{B}^*_{12} + B^*_{66})v^0_{,x\theta}/R$$
$$-(\bar{B}_{11} + \bar{D}_{11}/R)w^0_{,xxx} - (2B^*_{66} - D^*_{66} + \bar{B}^*_{12})w^0_{,x\theta\theta}/R + \bar{A}_{12}w^0_{,x}/R + P_1 = \rho_0 h\ddot{u}^0,$$
$$(\bar{A}_{12} + A_{66} + \bar{B}^*_{12} + B^*_{66})u^0_{,x\theta}/R + (A_{66} + 3B^*_{66} + 3D^*_{66})v^0_{,xx} + (\bar{A}_{22} + \bar{B}^*_{22})v^0_{,\theta\theta}/R^2$$
$$-(\bar{B}^*_{12} + 2B^*_{66} + \bar{D}^*_{12} + 3D^*_{66})w^0_{,xx\theta} - \bar{B}^*_{22}w^0_{,\theta\theta\theta}/R^2 + \bar{A}_{22}w^0_{,\theta}/R^2 + P_2 = \rho_0 h\ddot{v}^0,$$
$$(\bar{B}_{11} + \bar{D}_{11}/R)u^0_{,xxx} + (2B^*_{66} - D^*_{66} + \bar{B}^*_{12})u^0_{,x\theta\theta}/R - \bar{A}_{12}u^0_{,x}/R + (\bar{B}^*_{12} + 2B^*_{66}$$
$$+ \bar{D}^*_{12} + 3D^*_{66})v^0_{,xx\theta} + \bar{B}^*_{22}v^0_{,\theta\theta\theta}/R^2 - \bar{A}_{22}v^0_{,\theta}/R^2 - \bar{D}_{11}w^0_{,xxxx}$$
$$-2(D^*_{12} + 2D^*_{66})w^0_{,xx\theta\theta} - \bar{D}_{22}w^0_{,\theta\theta\theta\theta}/R^4 + 2\bar{B}^*_{12}w^0_{,xx} + 2(\bar{B}^*_{22} - \bar{D}^*_{22})w^0_{,\theta\theta}/R^2$$
$$-(\bar{A}_{22} - \bar{B}^*_{22} + \bar{D}^*_{22})w^0/R^2 + P_3 = \rho_0 h\ddot{w}^0, \tag{31}$$

with

$$P_1 = p_x + N^a_{x,x}, \quad P_2 = p_\theta + m_\theta/R + N^a_{\theta,\theta}/R + M^a_{\theta,\theta}/R^2,$$
$$P_3 = p_z - N^a_\theta/R + M^a_{\theta,\theta\theta}/R^2 + M^a_{x,xx} + m_{x,x} + m_{\theta,\theta}/R. \tag{32}$$

The boundary conditions at the simply supported ends are taken as

$$\text{at } x = 0, a\text{: } N_x = 0, \quad v^0 = w^0 = 0, \quad M_x = 0; \quad \text{at } \theta = 0, \psi\text{: } u^0 = w^0 = 0,$$
$$N_\theta = 0, \quad M_\theta = 0. \tag{33}$$

Using the expansions (18), the equations of motion (32) reduce to the discretized form of equations (19). The non-zero elements of the 3×3 symmetric matrices M,

K and vectors U, H for CST are given by

$$M_{11} = M_{22} = M_{33} = \rho_0 h, \quad K_{13} = -(\bar{B}_{11} + \bar{D}_{11}/R)\bar{m}^3$$

$$-(2B^*_{66} - D^*_{66} + \bar{B}^*_{12})\bar{m}\bar{n}^2/R - \bar{A}_{12}\bar{m}/R,$$

$$K_{11} = (\bar{A}_{11} + \bar{B}^*_{11})\bar{m}^2 + (A_{66} - B^*_{66} + D^*_{66})\bar{n}^2/R^2,$$

$$K_{12} = (\bar{A}_{12} + A_{66} + \bar{B}^*_{12} + B^*_{66})\bar{m}\bar{n}/R.$$

$$K_{22} = (A_{66} + 3B^*_{66} + 3D^*_{66})\bar{m}^2 + (\bar{A}_{22} + \bar{B}^*_{22})\bar{n}^2/R^2, \quad U = [u^0\ v^0\ w^0]^{\mathrm{T}}_{mn},$$

$$K_{23} = -(\bar{B}^*_{12} + 2B^*_{66} + \bar{D}^*_{12} + 3D^*_{66})\bar{m}^2\bar{n} - \bar{B}^*_{22}\bar{n}^3/R^2 - \bar{A}_{22}\bar{n}/R^2,$$

$$K_{33} = \bar{D}_{11}\bar{m}^4 + 2(D^*_{12} + 2D^*_{66})\bar{m}^2\bar{n}^2 + \bar{D}_{22}\bar{n}^4/R^4 + 2\bar{B}^*_{12}\bar{m}^2 + 2(\bar{B}^*_{22} - \bar{D}^*_{22})\bar{n}^2/R^2$$

$$+ (\bar{A}_{22} - \bar{B}^*_{22} + \bar{D}^*_{22})/R^2,$$

$$H_1 = \bar{m}(N^a_x)_{mn} + p_{x_{mn}}, \quad H_2 = [\bar{n}M^a_\theta/R^2 + \bar{n}N^a_\theta/R + p_\theta + m_\theta/R]_{mn},$$

$$H_3 = [p_z - N^a_\theta/R - \bar{n}^2 M^a_\theta M^t_\theta/R^2 - \bar{m}^2 M^a_x - \bar{m}m_x - \bar{n}m_\theta/R]_{mn}. \tag{34}$$

The three natural frequencies and mode shapes for classical theory predominantly correspond to two in-plane and one bending mode of vibration.

Proceeding as in FSDT, the modal sensitivity and actuation factors are obtained as

$$S^m_{mn} = h_s[\bar{m}e_{31}X^b_1 + e_{32}\bar{n}X^b_2/R - e_{32}\{1 + h^2_s/12(R + z_s)^2\}X^b_3/(R + z_s)]/(\pi^2 mn\eta_{33}), \tag{35}$$

$$S^b_{mn} = h_s[-\bar{m}^2 z_3 e_{31}X^b_3 + \bar{n}^2 e_{32}\{h^2_s/12(R + z_s)^3 - z_s/R(R + z_s)\}X^b_3]/(\pi^2 mn\eta_{33}), \tag{36}$$

$$A^m_{mn} = -\frac{4}{aR\psi}[(1 + z_a/R)(e_{31}RX^b_1/\bar{n} + e_{32}X^b_2/\bar{m}) - e_{32}X^b_3/\bar{m}\bar{n}], \tag{37}$$

$$A^b_{mn} = \frac{4}{aR\psi}[\{z_a(1 + z_a/R) + h^2_a/12R\}e_{31}R\bar{m}/\bar{n} + z_a e_{32}\bar{n}/R\bar{m}]X^b_3. \tag{38}$$

5. MODAL SENSITIVITY AND ACTUATION FACTORS FOR PLATES

Consider a simply supported, composite rectangular plate of sides a, b and thickness h (Figure 1(b)) with the co-ordinates x, y spanning its mid-plane. The results for composite plates are obtained as particular cases of those for shells using

FSDT and CLT, by replacing $\theta, \psi, \bar{n}$ with $y/R, b/R, \hat{n}R$, respectively, where $\hat{n} = n\pi/b$ and taking the limit as $R \to \infty$. Thus, we obtain for FSDT

$$\phi^{s(b)}_{mn_l} = \zeta(S^m_{mn} + S^b_{mn})(\cos \bar{m}x_{l_1} - \cos \bar{m}x_{l_2})(\cos \hat{n}y_{l_1} - \cos \hat{n}y_{l_2})ab/A^s_l, \tag{39}$$

where

$$S^m_{mn} = h_s(\bar{m}e_{31}X^b_1 + \hat{n}e_{32}X^b_2)/(\pi^2 mn\eta_{33}), \quad S^b_{mn} = h_s z_s(\bar{m}e_{31}X^b_4 + \hat{n}e_{32}X^b_5)/(\pi^2 mn\eta_{33}), \tag{40}$$

and for CLT

$$S^m_{mn} = h_s(\bar{m}e_{31}X^b_1 + \hat{n}e_{32}X^b_2)/(\pi^2 mn\eta_{33}), \quad S^b_{mn} = -h_s z_s(\bar{m}^2 e_{31} + \hat{n}^2 e_{32})X^b_3/(\pi^2 mn\eta_{33}). \tag{41}$$

The total modal force due to actuation for the (m, n)th bending mode of plates is expressed, using FSDT, as

$$F = \sum_{l=1}^{pq} F_l, \quad \text{with} \quad F_l = -A^t_{mn}\phi^l(\cos \bar{m}x_{l_1} - \cos \bar{m}x_{l_2})(\cos \hat{n}y_{l_1} - \cos \hat{n}y_{l_2}), \tag{42}$$

and

$$A^t_{mn} = A^m_{mn} + A^b_{mn}, \quad A^m_{mn} = -\frac{4}{ab}[e_{31}X^b_1/\hat{n} + e_{32}X^b_2/\bar{m}],$$

$$A^b_{mn} = -\frac{4}{ab}[\tfrac{1}{2}h_a(k_{55}\bar{m}e_{15}/\hat{n} + k_{44}\hat{n}e_{24}/\bar{m})X^b_3 + (z_a e_{31} + \tfrac{1}{2}h_a k_{55}e_{15})X^b_4/\hat{n}$$
$$+ (z_a e_{32} + \tfrac{1}{2}h_a k_{44}e_{24})X^b_5/\hat{m}]. \tag{43}$$

The actuation factors for the case of CLT are obtained as

$$A^m_{mn} = -\frac{4}{ab}[e_{31}X^b_1/\hat{n} + e_{32}X^b_2/\bar{m}], \quad A^b_{mn} = \frac{4}{ab}(z_a e_{31}\bar{m}/\hat{n} + z_a e_{32}\hat{n}/\bar{m})X^b_3. \tag{44}$$

The membrane actuation factor is the same as in FSDT.

6. CLOSED-LOOP VELOCITY FEEDBACK CONTROL

In the closed-loop velocity feedback control, the top piezoelectric layer serves as an actuator and the bottom one serves as a sensor. These are segmented into the same rectangular pattern such that each actuator patch has a corresponding sensor patch. Each sensor signal is processed and fed back to the corresponding collacated distributed actuator patch. The output signal of the lth sensor patch is contributed by the direct piezoelectric effect. In the velocity feedback (derivative feedback) control, the lth actuator voltage is proportional to the derivative of the collocated sensor signal. Imposing modal filters to isolate the (m, n)th bending mode signal, the modal feedback voltage is given by

$$\phi_l = G' \dot{\phi}^{s(b)}_{mn_l}, \tag{45}$$

where G' is the velocity feedback gain. Let G be the signal amplification factor, i.e.,

$$G = \text{(amplitude of feedback voltage)}/\text{(amplitude of sensing signal voltage)}.$$

It follows from equation (45) that the true modal velocity gain factor $G' = G/\omega$ where ω is the natural frequency in radians per second for that mode. The resultant modal control force from all actuator patches for the shell is obtained using equations (23), (34) and (45):

$$F(t) = -\dot{\zeta}(t) A^t_{mn} S^t_{mn} G' \sum_{l=1}^{pq} (\cos \bar{m} x_{l_1} - \cos \bar{m} x_{l_2})^2 (\cos \bar{n}\theta_{l_1} - \cos \bar{n}\theta_{l_2})^2 a R_1 \psi / A^s_l,$$

$$F(t) = -\dot{\zeta}(t) A^t_{mn} S^t_{mn} G' \Delta \tag{46}$$

with

$$\Delta = \sum_{l=1}^{pq} (\cos \bar{m} x_{l_1} - \cos \bar{m} x_{l_2})^2 (\cos \bar{n}\theta_{l_1} - \cos \bar{n}\theta_{l_2})^2 a R_1 \psi / A^s_l. \tag{47}$$

Δ denotes the spatial effect of the segmented actuator. Similarly, for the plate, $F(t)$ is given by equation (46) with

$$\Delta = \sum_{l=1}^{pq} (\cos \bar{m} x_{l_1} - \cos \bar{m} x_{l_2})^2 (\cos \hat{n} y_{l_1} - \cos \hat{n} y_{l_2})^2 ab / A^s_l. \tag{48}$$

The equation of motion (21) for the (m, n)th bending mode becomes

$$\ddot{\zeta} + (2\xi\omega + A^t_{mn} S^t_{mn} G' \Delta)\dot{\zeta} + \omega^2 \zeta = F^m, \tag{49}$$

where F^m is the mechanical load for this mode. The increase ξ^c in the modal damping ratio is given by

$$\xi^c = A^t_{mn} S^t_{mn} G \Delta / 2\omega^2 = (A^m_{mn} + A^b_{mn})(S^m_{mn} + S^b_{mn}) G \Delta / 2\omega^2. \tag{50}$$

7. CONCLUSIONS

The effect of transverse shear is significant for composite thin and thick plates and shells with fibre-reinforced composite elastic substrate, especially for higher modes due to the small ratio of the transverse shear modulus to the in-plane longitudinal Young's modulus. A Flugge-type first order shear deformation theory (FSDT) has been developed including the rotary inertia of the elastic plies and the inertia, stiffness and piezoelectric effects (direct and converse) of the piezoelectric layers. Plates and shells with any cross-ply lamination scheme (symmetric/unsymmetric) are considered. Analytical expressions for modal membrane and bending sensitivity factors, membrane and bending actuation factors and controlled damping ratio are developed for cylindrical shells and plates using FSDT and classical lamination theory. The contribution of the in-plane displacement components is included. These results are used in the second part of this study to illustrate the effect of the thickness parameter on the sensitivity factors, actuation factors, and controlled damping ratio.

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